Independent Schools Examinations Board

COMMON ENTRANCE EXAMINATION AT 13+

MATHEMATICS III

Thursday 28 February 2002

Please read this information before the examination starts.

- This examination is 60 minutes long.
- Answer as many questions as possible. They may be done in any order.
- **Failure to show necessary working may result in loss of marks.**
- Electronic calculators may be used in any question.
- Candidates are expected to give answers to an appropriate degree of accuracy.
- Solutions to questions which require accurate drawing should be done on graph or squared paper.

1. (a) Given that

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and $a = 3$ $b = {}^{-}4$ $c = {}^{-}8$ find the value of

(i) b^2 (1)

(ii) $4ac$ (1)

(iii) $\sqrt{b^2 - 4ac}$ (2)

(iv) x (2)

(b) When $x = 2$ and $y = 3$ find the value of

$$\frac{y}{x} \div \frac{x}{y}$$ (2)

(c) Factorise

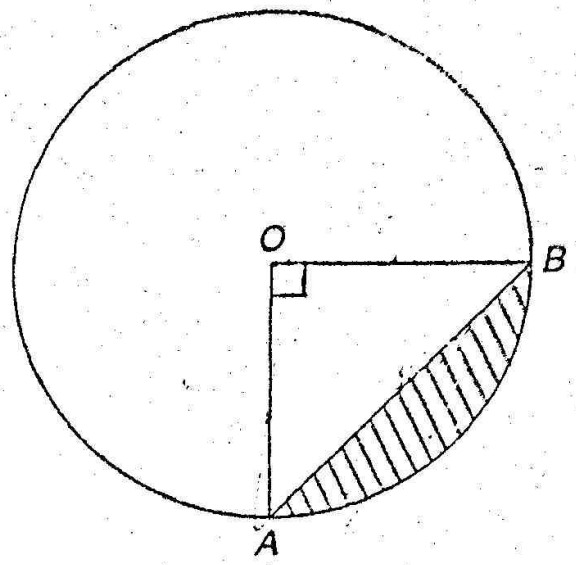
(i) $a^2 - 4ac$ (2)

(ii) $6x^3 + 2x$ (2)

(d) Solve the simultaneous equations

$$3p + q = 2$$
$$4p - 2q = 11$$ (5)

2. *O* is the centre of a circle of radius 8 cm.



not to scale

Calculate

(i) the length of the chord *AB* (3)

(ii) the area of the triangle *OAB* (2)

(iii) the perpendicular distance from *O* to the chord *AB* (4)

(iv) the area of the circle (2)

(v) the percentage of the area of the circle occupied by the triangle *OAB* (3)

(vi) the area of the shaded segment of the circle. (3)

3. (a) Each term in a series is given by the formula

$$t_n = n^2 - n + 1$$

so that $t_1 = 1^2 - 1 + 1 = 1$

(i) Calculate

(a) the 2nd term (1)

(b) the 3rd term (1)

(c) the 10th term (1)

(d) the 100th term in the series. (1)

(ii) Find the first term that is

(a) greater than 100 (1)

(b) greater than 1000 (3)

(b) The sum of the first n terms of a different series can be found by substituting the value of n into the formula

$$S_n = n^2 + 2n$$

e.g. S_4 = the sum of the first four terms = 16 + 8 = 24

Calculate

(i) S_1 = the first term of the series (1)

(ii) S_2 = the sum of the first two terms of the series (1)

(iii) the second term of the series (1)

(iv) the third term of the series (2)

(v) the nth term of the series. (3)

4. When Mr Ford travels to his school reunion, he likes to take the scenic route.

He drives for 2 hours and 15 minutes at an average speed of 40 miles per hour.

(i) Calculate the distance that he drives. (3)

Mr Ford makes just one stop at 11:00 hrs which lasts for 25 minutes.

(ii) If he arrives at his school reunion at 12:05 hrs, at what time does he begin his journey? (3)

Mrs Morris also travels to the school reunion, but without stopping for a break.

She travels a distance of 104 miles at an average speed of 60 miles per hour.

(iii) At what time should she leave home so as to arrive at the school reunion at the same time as Mr Ford? (4)

Mr Austin travels to the school reunion and takes 1 hour 36 minutes to travel a distance of 80 miles.

(iv) Calculate the average speed, in miles per hour, at which Mr Austin travels. (3)

4)

i) $D = S \times T = 40 \times 2.25 = 90$

ii) Total time = 2 hrs 40 mins

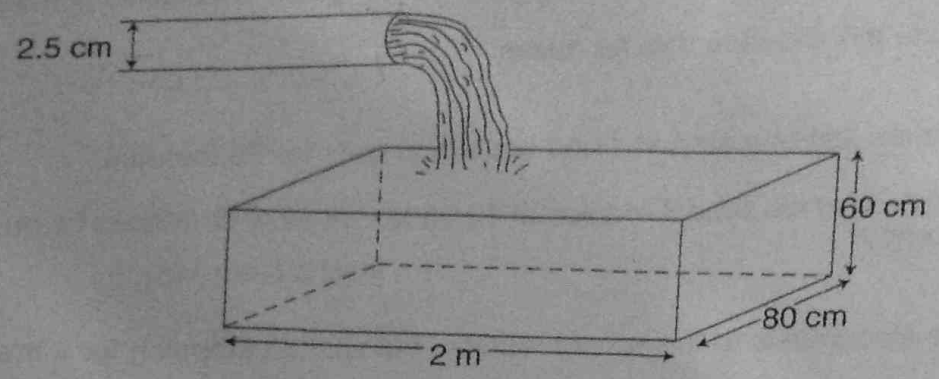
12:05 - 2 hrs 40 mins = 9:25

iii) $T = \frac{D}{S} = \frac{104}{60} = 1.7\dot{3}$ hours = 1 hr 44 mins

12:05 - 1 hr 44 mins = 10:21

iv) $45 \cdot \frac{TD}{TT} = \frac{80}{1.6} = 50$ mph

5. A cylindrical pipe delivers water via a small garden pump to an open trough which is in the shape of a cuboid.



not to scale

The trough measures 2 m by 80 cm and has a depth of 60 cm.

(i) Calculate the volume of the open trough, giving your answer in

(a) cubic centimetres 960,000 cm (3)

(b) litres. 960 (2)

1000cm³ = 1 Litre

The pipe has an internal diameter of 2.5 cm.

(ii) Calculate the internal cross-sectional area of the cylindrical pipe. $\pi r^2 = \pi(1.25)^2 = 4.91$ (3)

(iii) Calculate the volume of water in a 1 metre section of pipe, giving your answer in

(a) cubic centimetres $\pi(1.25)^2 \times 100 = 490.9$ (2)

(b) litres. 490.9 (1)

Water flows through the pipe at a rate of 2750 litres per hour.

(iv) Calculate the time taken to completely fill the open trough, giving your answer to the nearest minute. Need to fill 960 (2)

(v) Calculate how many litres of water flow through the pipe each second. (2)

(vi) Calculate the speed at which the water flows through the pipe, giving your answer in metres per second. (3)

iv) $\frac{960}{2750} = 0.349$ hours $= 20.94$ mins $= 21$ mins

v) 2750 litres per hour $= \frac{2750}{60 \times 60} = 0.764$ litres per second

vi) 2750 litres per hour = m³/sec

Hand in this tear-off sheet with your other answers.

SURNAME ..... FIRST NAME .....
(Block capitals please)
JUNIOR SCHOOL ..... SENIOR SCHOOL .....

6. **All rectangles in this question have an area of 24 cm^2**

One example is a 1 cm by 24 cm rectangle.

(i) Three other rectangles have dimensions which are different integers. Write down these dimensions, noting that 1 cm by 24 cm and 24 cm by 1 cm count as the same.

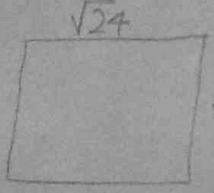
Answers: 4 cm by 6 cm

8 cm by 3 cm

2 cm by 12 cm (3)

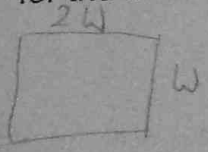
Some dimensions are not integers, for example 3.2 cm by 7.5 cm.

(ii) One special rectangle has equal sides. Calculate the length of each side.



Answer: 4.898979 or $\sqrt{24}$ cm (2)

(iii) (a) If the length of a rectangle is twice its width, write down an expression for the area of the rectangle in terms of its width, w.



Answer: Area = $2w^2$ (2)

(b) Show that $w^2 = 12$

$2w^2 = 24$
$w^2 = 12$

(2)

(c) Calculate the dimensions of the rectangle in part (iii) (a).

$w = \sqrt{12}$

Answers: $\sqrt{12}$ cm by $2\sqrt{12}$ cm (2)

(iv) Complete the table of values for the rectangles described in the table.

width	length in terms of width	length in terms of w	equation for the area of rectangle	value of w (cm)	length (cm)	perimeter of the rectangle (cm)
w	equal to width					
w	twice width					
w	3 times width	$3w$	$3w^2 = 24$	2.83	8.49	22.6
w	4 times width					
w	6 times width					

(7)

(v) Comment on the relationship between the perimeter and the shape of the rectangles as the dimensions change.

Answer: .....

..... (1)

(Total marks: 100)